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A Model Independent Determination of $|V_{ub}|$

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Abstract

It is shown that measuring the lepton invariant mass spectrum in inclusive semileptonic $\bar{B} \to X_u \ell \bar{\nu}$ decay yields a model independent determination of $|V_{ub}|$. Unlike the lepton energy and hadronic invariant mass spectra, nonperturbative effects are only important in the resonance region, and play a parametrically suppressed role when $\mathrm{d}\Gamma/\mathrm{d}q^2$ is integrated over $q^2 > (m_B - m_D)^2$, which is required to eliminate the charm background. Perturbative and nonperturbative corrections are presented to order $\alpha_s^2 \beta_0$ and $\Lambda_{\mathrm{QCD}}^2/m_b^2$, and the $\Lambda_{\mathrm{QCD}}^3/m_b^3$ corrections are used to estimate the uncertainty in our results. The utility of the $\bar{B} \to X_s \ell^+ \ell^-$ decay rate above the $\psi(2S)$ resonance is discussed.

A precise and model independent determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix element V_{ub} is important for testing the Standard Model at B factories via the comparison of the angles and the sides of the unitarity triangle. The first extraction of $|V_{ub}|$ from experimental data relied on a study of the lepton energy spectrum in inclusive charmless semileptonic B decay [1]. Recently $|V_{ub}|$ was also measured from exclusive semileptonic $\bar{B} \to \rho \ell \bar{\nu}$ and $\bar{B} \to \pi \ell \bar{\nu}$ decay [2], and from inclusive decays using the reconstruction of the invariant mass of the hadronic final state [3].

These determinations suffer from large model dependence. The exclusive $|V_{ub}|$ measurements rely on form factor models or quenched lattice calculations at the present time.[1] Inclusive B decay rates are currently on a better theoretical footing, since they can be computed model independently in a series in $\Lambda_{\rm QCD}/m_b$ and $\alpha_s(m_b)$ using an operator product expansion (OPE) [7–10]. However, the predictions of the OPE are only model independent for sufficiently inclusive observables, while the $\bar{B} \to X_u \ell \bar{\nu}$ decay rate can only be measured by imposing severe cuts on the phase space to eliminate the ~ 100 times larger $\bar{B} \to X_c \ell \bar{\nu}$ background. For both the charged lepton and hadronic invariant mass spectra, these cuts spoil the convergence of the OPE, and the most singular terms must be resummed into a nonperturbative b quark distribution function [11]. While it may be possible to extract this from the photon spectrum in $B \to X_s \gamma$ [11,12], it would clearly be simpler to find an observable for which the OPE did not break down in the region of phase space free from charm background. In this Letter we show that this is the situation for the lepton invariant mass spectrum.

At leading order in the $\Lambda_{\rm QCD}/m_b$ expansion the B meson decay rate is equal to the b quark decay rate. Nonperturbative effects are suppressed by at least two powers of $\Lambda_{\rm QCD}/m_b$. Corrections of order $\Lambda_{\rm QCD}^2/m_b^2$ are characterized by two heavy quark effective theory (HQET) matrix elements [8–10], which are defined by

$$\begin{aligned}
\lambda_1 &= \langle B(v)|\, \bar{h}_v^{(b)}\, (iD)^2\, h_v^{(b)}\, |B(v)\rangle / 2m_B\,, \\
\lambda_2 &= \langle B(v)|\, \frac{g_s}{2}\, \bar{h}_v^{(b)}\, \sigma_{\mu\nu} G^{\mu\nu}\, h_v^{(b)}\, |B(v)\rangle / 6m_B\,.
\end{aligned} \tag{1}$$

These matrix elements also occur in the expansion of the B and B^* masses in powers of $\Lambda_{\rm QCD}/m_b$,

$$m_B = m_b + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_b} + \ldots\,, \qquad m_{B^*} = m_b + \bar{\Lambda} - \frac{\lambda_1 - \lambda_2}{2m_b} + \ldots\,. \tag{2}$$

Similar formulae hold for the D and D^* masses. The parameters $\bar{\Lambda}$ and λ_1 are independent of the heavy b quark mass, while there is a weak logarithmic scale dependence in λ_2. The measured $B^* - B$ mass splitting fixes $\lambda_2(m_b) = 0.12\,{\rm GeV}^2$. At $\mathcal{O}(\Lambda_{\rm QCD}^3/m_b^3)$ seven additional

[1] A model independent determination of $|V_{ub}|$ from exclusive decays is possible without first order heavy quark symmetry or chiral symmetry breaking corrections [4], but it requires data on $\bar{B} \to K^* \ell^+ \ell^-$. A model independent extraction is also possible from decays to wrong-sign charm [5], but this is very challenging experimentally. See also [6] for a discussion of extracting $|V_{ub}|$ from a comparison of photon spectra in B and D radiative leptonic decays.

parameters arise in the OPE [13–15], and varying these parameters is often used to estimate the theoretical uncertainty in the OPE [14–16].

In inclusive semileptonic B decay, for a particular hadronic final state X, the maximum lepton energy is $E_\ell^{(\max)} = (m_B^2 - m_X^2)/2m_B$ (in the B rest frame), so to eliminate charm background one must impose a cut $E_\ell > (m_B^2 - m_D^2)/2m_B$. The maximum lepton energy in semileptonic b quark decay is $m_b/2$, which is less than the physical endpoint $m_B/2$. Their difference, $\bar{\Lambda}/2$, is comparable in size to the endpoint region $\Delta E_\ell^{(\mathrm{endpoint})} = m_D^2/2m_B \simeq 0.33\,\mathrm{GeV}$. The effects which extend the lepton spectrum beyond its partonic endpoint appear as singular terms in the prediction for $\mathrm{d}\Gamma/\mathrm{d}E_\ell$ involving derivatives of delta functions, $\delta^{(n)}(1-2E_\ell/m_b)$. The lepton spectrum must be smeared over a region of energies ΔE_ℓ near the endpoint before theory can be compared with experiment. If the smearing region ΔE_ℓ is much smaller than Λ_{QCD}, then higher dimension operators in the OPE become successively more important and the OPE is not useful for describing the lepton energy spectrum. For $\Delta E_\ell \gg \Lambda_{\mathrm{QCD}}$, higher dimension operators become successively less important and a useful prediction for the lepton spectrum can be made using the first few terms in the OPE. When $\Delta E_\ell \sim \Lambda_{\mathrm{QCD}}$, there is an infinite series of terms in the OPE which are all equally important. Since $\Delta E_\ell^{(\mathrm{endpoint})}$ is about Λ_{QCD}, it seems unlikely that predictions based on a few low dimension operators in the OPE can successfully determine the lepton spectrum in this region.

It was shown in [11] that the leading singularities in the OPE may be resummed into a nonperturbative light-cone distribution function $f(k_+)$ for the heavy quark. To leading order in $1/m_b$, the effects of the distribution function may be included by replacing m_b by $m_b^* \equiv m_b + k_+$, and integrating over the light-cone momentum

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\ell} = \int \mathrm{d}k_+\, f(k_+)\, \frac{\mathrm{d}\Gamma_{\mathrm{p}}}{\mathrm{d}E_\ell}\bigg|_{m_b \to m_b^*}\,, \tag{3}$$

where $\mathrm{d}\Gamma_{\mathrm{p}}/\mathrm{d}E_\ell$ is the parton-level spectrum. Analogous formulae hold for other differential distributions [17,18]. For purposes of illustration, we will use a simple model for the structure function given by the one-parameter ansatz [19]

$$f(k_+) = \frac{32}{\pi^2\Lambda}\,(1-x)^2\, \exp\left[-\frac{4}{\pi}(1-x)^2\right] \Theta(1-x)\,, \qquad x \equiv \frac{k_+}{\Lambda}\,, \tag{4}$$

taking the model parameter $\Lambda = 0.48\,\mathrm{GeV}$, corresponding to $m_b = 4.8\,\mathrm{GeV}$.

The charm background can also be eliminated by reconstructing the invariant mass of the hadronic final state, m_X, since decays with $m_X < m_D$ must arise from $b \to u$ transition. While this analysis is challenging experimentally, the $m_X < m_D$ cut allows a much larger fraction of $b \to u$ decays than the $E_\ell > (m_B^2 - m_D^2)/2m_B$ constraint. This is expected to result in a reduction of the theoretical uncertainties [20,21], although both the lepton endpoint region, $E_\ell > (m_B^2 - m_D^2)/2m_B$, and the low hadronic invariant mass region, $m_X < m_D$, receive contributions from the same set of hadronic final states (but with very different weights). However, the same nonperturbative effects which lead to the breakdown of predictive power in the lepton endpoint region also give large uncertainties in the hadron mass spectrum over the range $m_X^2 \sim \bar{\Lambda} m_b$ [20]. In other words, nonperturbative effects

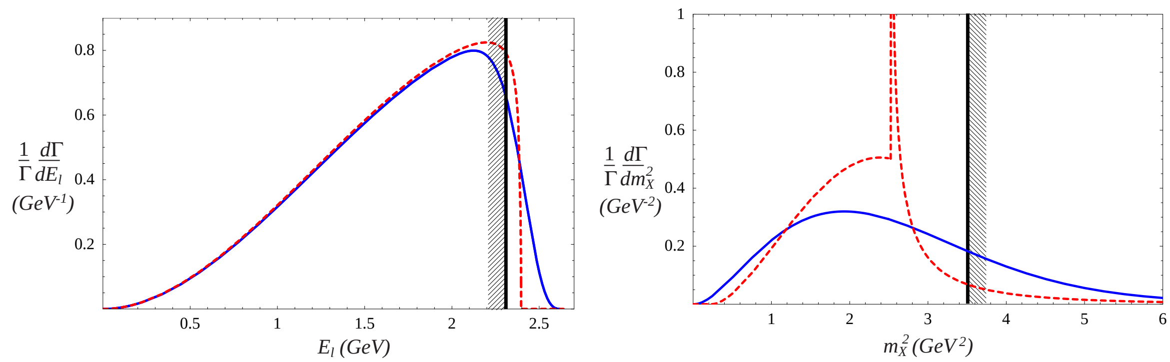

FIG. 1. The lepton energy and hadronic invariant mass spectra. The dashed curves are the b quark decay results to $\mathcal{O}(\alpha_s)$, while the solid curves are obtained by smearing with the model distribution function $f(k_+)$ in Eq. (4). The unshaded side of the vertical lines indicate the region free from charm background. The area under each curve has been normalized to one.

yield formally $\mathcal{O}(1)$ uncertainties in both cases, because numerically $m_D^2 \sim \Lambda_{\rm QCD} m_B$. The situation is illustrated in Fig. 1.

The situation is very different for the lepton invariant mass spectrum. Decays with $q^2 \equiv (p_\ell + p_{\bar\nu})^2 > (m_B - m_D)^2$ must arise from $b \to u$ transition. Such a cut forbids the hadronic final state from moving fast in the B rest frame, and so the light-cone expansion which gives rise to the shape function is not relevant in this region of phase space.[2] This is clear from the kinematics: the difference between the partonic and hadronic values of maximum q^2 is $m_B^2 - m_b^2 \sim 2\bar\Lambda\, m_b$, and nonperturbative effects are only important in a region of comparable size. For example, the most singular term in the OPE at order $(\Lambda_{\rm QCD}/m_b)^3$ is of order $(\Lambda_{\rm QCD}/m_b)^3\ \delta(1 - q^2/m_b^2)$. This contribution to the decay rate is not suppressed compared to the lowest order term in the OPE only if the spectrum is integrated over a small region of width $\Delta q^2 \sim \Lambda_{\rm QCD}\, m_b$ near the endpoint. This is the resonance region where only hadronic final states with masses $m_X \sim \Lambda_{\rm QCD}$ can contribute, and the OPE is not expected to work anyway. In contrast, nonperturbative effects are important in the E_ℓ and m_X^2 spectra in a parametrically much larger region where final states with masses $m_X^2 \sim \Lambda_{\rm QCD}\, m_b$ contribute.[3] The better behavior of the q^2 spectrum than the E_ℓ and m_X^2 spectra is also reflected in the perturbation series. There are Sudakov double logarithms near the phase space boundaries in the E_ℓ and m_X^2 spectra, whereas there are only single logarithms in the q^2 spectrum.

The effect of smearing the q^2 spectrum with the model distribution function in Eq. (4) is

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[2]The fact that the b quark distribution function is not relevant for large q^2 was pointed out in [22] in the context of $\bar B \to X_s \ell^+\ell^-$ decay and in [18] for semileptonic $\bar B \to X_u$ decay.

[3]Similar arguments also show that the light-cone distribution function is not relevant for small hadron energy, E_X, but it does enter for E_X near $m_b/2$. If $m_b/2 - m_c \gg \Lambda_{\rm QCD}$, then the constraint $E_X < m_D$ in the B rest frame would also give a model independent determination of $|V_{ub}|$.

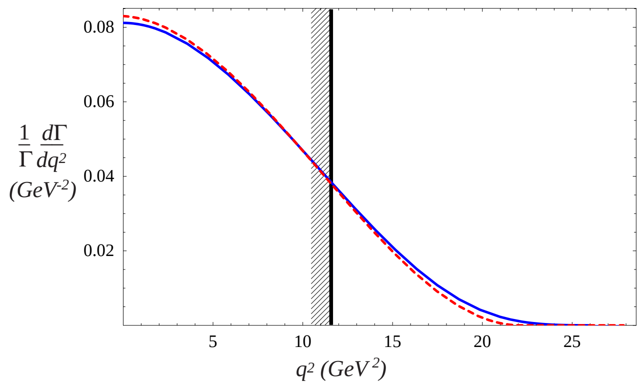

FIG. 2. The lepton invariant mass spectrum to $\mathcal{O}(\alpha_s)$. The meaning of the curves is the same as in Fig. 1.

Decay distribution	Width of region without charm background	Nonperturbative region near endpoint	Fraction of $b \to u$ events included
$\mathrm{d}\Gamma/\mathrm{d}E_\ell$	$\Delta E_\ell = m_D^2/2m_B$	$\Delta E_\ell \sim \Lambda_{\rm QCD}$	$\sim 10\%$
$\mathrm{d}\Gamma/\mathrm{d}m_X^2$	$\Delta m_X^2 = m_D^2$	$\Delta m_X^2 \sim \Lambda_{\rm QCD}\, m_b$	$\sim 80\%$
$\mathrm{d}\Gamma/\mathrm{d}q^2$	$\Delta q^2 = 2m_B m_D - m_D^2$	$\Delta q^2 \sim \Lambda_{\rm QCD}\, m_b$	$\sim 20\%$

TABLE I. Comparison between the lepton energy, hadronic invariant mass, and lepton invariant mass spectra for the determination of $|V_{ub}|$. The region dominated by nonperturbative effects is parametrically smaller than the region without charm background only for the q^2 spectrum in the last row (viewing $m_D^2 \sim m_c^2 \sim \Lambda_{\rm QCD}\, m_b$). The last column gives rough numbers corresponding to the plots in Figs. 1 and 2.

illustrated in Fig. 2. In accord with our previous arguments, it is easily seen to be subleading over the region of interest. Table I compares qualitatively the utility of the lepton energy, the hadronic invariant mass, and the lepton invariant mass spectra for the determination of $|V_{ub}|$.

We now proceed to calculate the $\bar B \to X_u \ell \bar\nu$ decay rate with lepton invariant mass above a given cutoff, working to a fixed order in the OPE (i.e., ignoring the light-cone distribution function which is irrelevant for our analysis). The lepton invariant mass spectrum including the leading perturbative and nonperturbative corrections is given by

$$\frac{1}{\Gamma_0}\,\frac{\mathrm{d}\Gamma}{\mathrm{d}\hat q^2} = \left(1+\frac{\lambda_1}{2m_b^2}\right) 2\,(1-\hat q^2)^2\,(1+2\hat q^2) + \frac{\lambda_2}{m_b^2}\,(3-45\hat q^4+30\hat q^6) + \frac{\alpha_s(m_b)}{\pi}\,X(\hat q^2) + \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \beta_0\,Y(\hat q^2) + \ldots\,, \tag{5}$$

where $\hat q^2 = q^2/m_b^2$, $\beta_0 = 11 - 2n_f/3$, and

$$\Gamma_0 = \frac{G_F^2\,|V_{ub}|^2\,m_b^5}{192\,\pi^3} \tag{6}$$

is the tree level $b \to u$ decay rate. The ellipses in Eq. (5) denote terms of order $(\Lambda_{\rm QCD}/m_b)^3$ and order α_s^2 terms not enhanced by β_0. The function $X(\hat q^2)$ is given analytically in Ref. [23],

whereas $Y(\hat{q}^2)$ was computed numerically in Ref. [24]. The order $1/m_b^3$ nonperturbative corrections were computed in Ref. [15]. The matrix element of the kinetic energy operator, λ_1, only enters the $\hat{q}^2$ spectrum in a very simple form, because the unit operator and the kinetic energy operator are related by reparameterization invariance [25]. Any quantity which can be written independent of the heavy quark velocity v must depend only on the combination $(1 + \lambda_1/2m_b^2)$. The $\hat{q}^2$ spectrum (and the total rate written in terms of m_b) are invariant under a redefinition of v, but, for example, the lepton energy spectrum is not since $E_\ell = v \cdot p_e$. (Equivalently, the λ_1 term is a time-dilation effect, and hence is universal in any quantity that is independent of the rest frame of the B meson [8].)

We shall compute the fraction of $\bar{B} \to X_u \ell \bar{\nu}$ events with $q^2 > q_0^2$, $F(q_0^2)$, as the relation between the total $\bar{B} \to X_u \ell \bar{\nu}$ decay rate and $|V_{ub}|$ has been extensively discussed in the literature [26,27], and is known including the full α_s^2 corrections [28]. After integrating the spectrum in Eq. (5), we can eliminate the dependence on the b quark mass in favor of the spin averaged meson mass $\bar{m}_B = (m_B + 3m_{B^*})/4 \simeq 5.313\,\text{GeV}$, following [29]. We find

$$
\begin{aligned}
F(q_0^2) = {} & 1 - 2Q_0^2 + 2Q_0^6 - Q_0^8 - \frac{4\bar{\Lambda}}{\bar{m}_B}\left(Q_0^2 - 3Q_0^6 + 2Q_0^8\right) - \frac{6\bar{\Lambda}^2}{\bar{m}_B^2}\left(Q_0^2 - 7Q_0^6 + 6Q_0^8\right) \\
& + \frac{2\lambda_1}{\bar{m}_B^2}\left(Q_0^2 - 3Q_0^6 + 2Q_0^8\right) - \frac{12\lambda_2}{\bar{m}_B^2}\left(Q_0^2 - 2Q_0^6 + Q_0^8\right) \\
& + \frac{\alpha_s(m_b)}{\pi}\,\tilde{X}(Q_0^2) + \left(\frac{\alpha_s(m_b)}{\pi}\right)^2 \beta_0\,\tilde{Y}(Q_0^2) + \ldots\,,
\end{aligned}
\tag{7}
$$

where $Q_0 \equiv q_0/\bar{m}_B$. The functions $\tilde{X}(Q_0^2)$ and $\tilde{Y}(Q_0^2)$ can be calculated from $X(\hat{q}^2)$ and $Y(\hat{q}^2)$. Converting to the physical B meson mass has introduced a strong dependence on the parameter $\bar{\Lambda}$, the mass of the light degrees of freedom in the B meson. For $q_0^2 = (m_B - m_D)^2 \simeq 11.6\,\text{GeV}^2$, we find $F(11.6\,\text{GeV}^2) = 0.287 + 0.027\alpha_s(m_b) - 0.016\alpha_s^2(m_b)\beta_0 - 0.20\bar{\Lambda}/(1\text{ GeV}) - 0.02\bar{\Lambda}^2/(1\text{ GeV}^2) + 0.02\lambda_1/(1\text{ GeV}^2) - 0.13\lambda_2/(1\text{ GeV}^2) + \ldots$. The order $\alpha_s\bar{\Lambda}$ term is negligible and has been omitted. Using $\bar{\Lambda} = 0.4\,\text{GeV}$, $\lambda_1 = -0.2\,\text{GeV}^2$ [30] and $\alpha_s(m_b) = 0.22$, we obtain $F(11.6\,\text{GeV}^2) = 0.186$. There are several sources of uncertainties in the value for F. The perturbative uncertainties are negligible, as can be seen from the size of the $\mathcal{O}(\alpha_s^2\beta_0)$ contributions. At the present time there is a sizable uncertainty since $\bar{\Lambda}$ is not known accurately. In the future, a $\pm 50\,\text{MeV}$ error in $\bar{\Lambda}$ will result in a $\pm 5\%$ uncertainty in F. Finally, uncertainties from $1/m_b^3$ operators can be estimated by varying the matrix elements of the dimension six operators within the range expected by dimensional analysis, as discussed in detail in [14–16]. This results in an additional $\pm 4\%$ uncertainty in F. We note that this is a somewhat ad hoc procedure, since there is no real way to quantify the theoretical error due to unknown higher order terms. Therefore, these estimates should be treated as nothing more than (hopefully) educated guesses. They do allow, however, for a consistent comparison of the uncertainties in different quantities.

If q_0^2 has to be chosen larger, then the uncertainties increase. For example, for $q_0^2 = 15\,\text{GeV}^2$, we obtain $F(15\,\text{GeV}^2) = 0.158 + 0.024\alpha_s(m_b) - 0.012\alpha_s^2(m_b)\beta_0 - 0.18\bar{\Lambda}/(1\text{ GeV}) + 0.01\bar{\Lambda}^2/(1\text{ GeV}^2) + 0.02\lambda_1/(1\text{ GeV}^2) - 0.13\lambda_2/(1\text{ GeV}^2) + \ldots \simeq 0.067$, using the previous values of $\bar{\Lambda}$ and λ_1. The perturbative uncertainties are still negligible, while the uncertainty due to a $\pm 50\,\text{MeV}$ error in $\bar{\Lambda}$ and unknown dimension six matrix elements increase to $\pm 14\%$ and $\pm 13\%$, respectively. (This uncertainty may be reduced using data on the rare decay

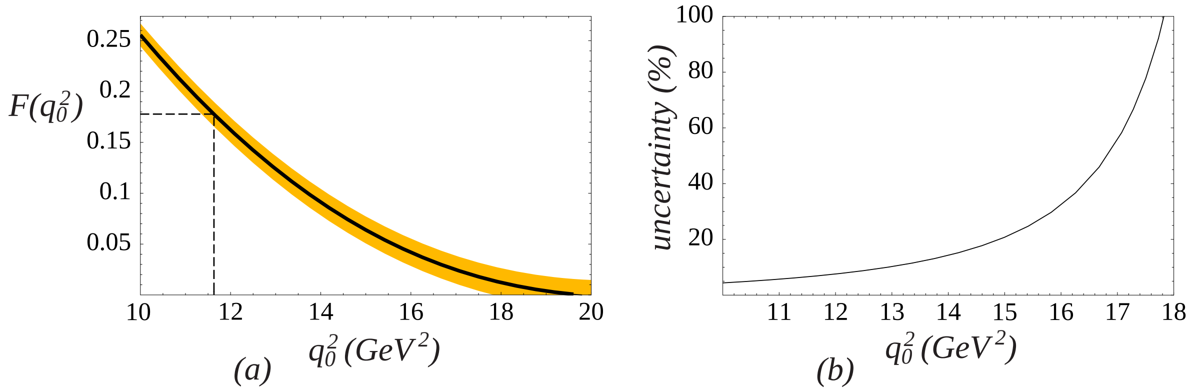

FIG. 3. (a) The fraction of $\bar{B} \to X_u \ell \bar{\nu}$ events with $q^2 > q_0^2$, $F(q_0^2)$, to order $\epsilon_{\rm BLM}^2$ and $\Lambda_{\rm QCD}^2/m_b^2$ in the upsilon expansion. The shaded region is the uncertainty due to $\Lambda_{\rm QCD}^3/m_b^3$ terms, as discussed in the text. The dashed line indicates the lower cut $q_0^2 = (m_B - m_D)^2 \simeq 11.6\,{\rm GeV}^2$, which corresponds to $F = 0.178 \pm 0.012$. (b) The estimated uncertainty in $F(q_0^2)$ due to $\Lambda_{\rm QCD}^3/m_b^3$ terms as a percentage of $F(q_0^2)$.

$\bar{B} \to X_s \ell^+ \ell^-$ in the large q^2 region, as discussed below.)

Another possible method to compute $F(q_0^2)$ uses the upsilon expansion [26]. By expressing $\hat{q}^2$ in terms of the Υ mass instead of $\bar{m}_B$, the dependence of $F(q_0^2)$ on $\bar{\Lambda}$ and λ_1 is eliminated. Instead, the result is sensitive to unknown nonperturbative contributions to m_Υ. The uncertainty related to these effects can be systematically taken into account and has been estimated to be small [26]. One finds,

$$|V_{ub}| = (3.04 \pm 0.06 \pm 0.08) \times 10^{-3} \left(\frac{\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})|_{q^2 > q_0^2}}{0.001 \times F(q_0^2)} \, \frac{1.6\,{\rm ps}}{\tau_B} \right)^{1/2} . \tag{8}$$

The errors explicitly shown in Eq. (8) are the estimates of the perturbative and nonperturbative uncertainties in the upsilon expansion, respectively.

For $q_0^2 = (m_B - m_D)^2$ we find $F(11.6\,{\rm GeV}^2) = 0.168 + 0.016\epsilon + 0.014\epsilon_{\rm BLM}^2 - 0.17\lambda_2 + \ldots \simeq 0.178$, where $\epsilon \equiv 1$ shows the order in the upsilon expansion. This result is in good agreement with 0.186 obtained from Eq. (7). The uncertainty due to $\bar{\Lambda}$ is absent in the upsilon expansion, however the size of the perturbative corrections has increased. The uncertainties due to $1/m_b^3$ operators is estimated to be $\pm 7\%$. For $q_0^2 = 15\,{\rm GeV}^2$, we obtain $F(15\,{\rm GeV}^2) = 0.060 + 0.011\epsilon + 0.011\epsilon_{\rm BLM}^2 - 0.14\lambda_2 + \ldots \simeq 0.064$, which is again in good agreement with 0.067 obtained earlier. For this value of q_0^2, the $1/m_b^3$ uncertainties increase to $\pm 21\%$. $F(q_0^2)$ calculated in the upsilon expansion is plotted in Fig. 3, where the shaded region shows our estimate of the uncertainty due to the $1/m_b^3$ corrections.

Concerning experimental considerations, measuring the q^2 spectrum requires reconstruction of the neutrino four-momentum, just like measuring the hadronic invariant mass spectrum. Imposing a lepton energy cut, which may be required for this technique, is not a problem. The constraint $q^2 > (m_B - m_D)^2$ automatically implies $E_\ell > (m_B - m_D)^2/2m_B \sim 1.1\,{\rm GeV}$ in the B rest frame. Even if the E_ℓ cut has to be slightly larger than this, the utility of our method will not be affected, but a dedicated calculation including the affects of arbitrary E_ℓ and q^2 cuts may be warranted.

If experimental resolution on the reconstruction of the neutrino momentum necessitates a significantly larger cut than $q_0^2 = (m_B - m_D)^2$, then the uncertainties in the OPE calculation of $F(q_0^2)$ increase. In this case, it may instead be possible to obtain useful model independent information on the q^2 spectrum in the region $q^2 > m_{\psi(2S)}^2 \simeq 13.6\,\mathrm{GeV}^2$ from the q^2 spectrum in the rare decay $\bar{B} \to X_s \ell^+ \ell^-$, which may be measured in the upcoming Tevatron Run-II. There are four contributions to this decay rate, proportional to the combination of Wilson coefficients $\widetilde{C}_9^2$, C_{10}^2, $C_7 \widetilde{C}_9$, and C_7^2. $\widetilde{C}_9$ is a q^2-dependent effective coefficient which takes into account the contribution of the four-quark operators. Its $\hat{q}^2$-dependence yields negligible uncertainties if we use a mean $\widetilde{C}_9$ obtained by averaging it in the region $0.5 < \hat{q}^2 < 1$ weighted with the b quark decay rate $(1-\hat{q}^2)^2\,(1+2\hat{q}^2)$. The resulting numerical values of the Wilson coefficients are $\widetilde{C}_9 = 4.47 + 0.44\,i$, $C_{10} = -4.62$, and $C_7 = -0.31$, corresponding to the scale $\mu = m_b$. In the $q^2 > m_{\psi(2S)}^2$ region the C_7^2 contribution is negligible, and the $C_7 \widetilde{C}_9$ term makes about a 20% contribution to the rate. For the $\widetilde{C}_9^2 + C_{10}^2$ contributions nonperturbative effects are identical to those which occur in $\bar{B} \to X_u \ell \bar{\nu}$ decay, up to corrections suppressed by $|\widetilde{C}_9 + C_{10}|/|\widetilde{C}_9 - C_{10}| \sim 0.02$. Therefore, the relation

$$\frac{\mathrm{d}\Gamma(\bar{B} \to X_u \ell \bar{\nu})/\mathrm{d}\hat{q}^2}{\mathrm{d}\Gamma(\bar{B} \to X_s \ell^+ \ell^-)/\mathrm{d}\hat{q}^2} = \frac{|V_{ub}|^2}{|V_{ts} V_{tb}|^2}\, \frac{8\pi^2}{\alpha^2}\, \frac{1}{|\widetilde{C}_9|^2 + |C_{10}|^2 + 12\,\mathrm{Re}\,(C_7 \widetilde{C}_9)/(1+2\hat{q}^2)}\,, \tag{9}$$

is expected to hold to a very good accuracy. There are several sources of corrections to this formula which need to be estimated: i) nonperturbative effects that enter the $C_7 \widetilde{C}_9$ term differently, ii) mass effects from the strange quark and muon, iii) higher $c\bar{c}$ resonance contributions in $\bar{B} \to X_s \ell^+ \ell^-$, and iv) scale dependence. Of these, i) and ii) are expected to be small unless q^2 is very close to m_B^2. The effects of iii) have also been estimated to be at the few percent level [22], although these uncertainties are very hard to quantify and could be comparable to the $\pm 8\%$ scale dependence [31] of the $\bar{B} \to X_s \ell^+ \ell^-$ rate. Integrating over a large enough range of q^2, $q_0^2 < q^2 < m_B^2$ with $m_{\psi(2S)}^2 < q_0^2 \lesssim 17\,\mathrm{GeV}^2$, the result implied by Eq. (9),

$$\frac{\mathcal{B}(\bar{B} \to X_u \ell \bar{\nu})|_{q^2 > q_0^2}}{\mathcal{B}(\bar{B} \to X_s \ell^+ \ell^-)|_{q^2 > q_0^2}} = \frac{|V_{ub}|^2}{|V_{ts} V_{tb}|^2}\, \frac{8\pi^2}{\alpha^2}\, \frac{1}{|\widetilde{C}_9|^2 + |C_{10}|^2 + 12\,\mathrm{Re}\,(C_7 \widetilde{C}_9)\, B(q_0^2)}\,, \tag{10}$$

is expected to hold at the $\sim 15\%$ level. Here $Q_0 \equiv q_0/\bar{m}_B$, and $B(q_0^2) = 2/[3(1+Q_0^2)] - 4(\bar{\Lambda}/\bar{m}_B)\, Q_0^2/[3(1+Q_0^2)^2] + \ldots$. For q_0^2 significantly above $(m_B - m_D)^2$, this formula may lead to a determination of $|V_{ub}|$ with smaller theoretical uncertainty than the one using the OPE calculation of $F(q_0^2)$.

In conclusion, we have shown that the q^2 spectrum in inclusive semileptonic $\bar{B} \to X_u \ell \bar{\nu}$ decay gives a model independent determination of $|V_{ub}|$ with small theoretical uncertainty. Nonperturbative effects are only important in the resonance region, and play a parametrically suppressed role when $\mathrm{d}\Gamma/\mathrm{d}q^2$ is integrated over $q^2 > (m_B - m_D)^2$, which is required to eliminate the charm background. This is a qualitatively better situation than the extraction of $|V_{ub}|$ from the endpoint region of the lepton energy spectrum, or from the hadronic invariant mass spectrum.

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